LITERATURE CITED

1. V. I. Shatrov and V. I. Yakovlev, "Change in the hydrodynamic drag of a sphere set in motion by electromagnetic forces," Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1981).
2. C. L. Lin and S. C. Lee, "Transient state analysis of separated flow around a sphere," Comput. Fluids, 1, No. 3 (1973).

LOCALLY THREE-DIMENSIONAL LAMINAR FLOWS

V. V. Bogolepov and I. I. Lipatov

UDC 532.526

Local variations in the flight vehicle surface, either specially designed or natural, may significantly affect heat transfer and skin friction and determine the state of the boundary layer. Analysis of the limiting solutions to Navier–Stokes equations as $\mathrm{Re} \to \infty$ (Re is Reynolds number) carried out in [1, 2] showed that different states of laminar boundary layer are possible near two-dimensional roughnesses, characterized by a difference in the ratio of viscous forces to inertia forces and in the nature of viscous-inviscid interaction near the roughness. The method of matched asymptotic expansions was used in [1, 2] to study such flow situations and numerical results for the corresponding boundary-value problem were obtained. Subsequently, results were obtained for studies on specific flow conditions or roughness shapes [3-9]. In practice, three-dimensional and not two-dimensional roughness is more frequently encountered; interest in the study of flow past such roughness is also associated with the problem of the flow past elements of relief on the earth's surface. Results of investigations on flow past three-dimensional roughness are given in [10-16]. However, not all possible flow conditions near three-dimensional roughness have been investigated. This paper deals with studies on the flow past roughness whose length is less or equal to the boundary-layer thickness as well as longer roughness in whose neighborhood there is no interaction with the external inviscid flow.

1. Consider a steady flow past three-dimensional roughness located at the bottom of a laminar boundary layer at a distance l from the leading edge of a flat (Fig. 1).

The coordinate system is chosen such that x axis is in the direction of the external flow, y axis is normal to the surface, and z axis is perpendicular to x and y axes. It is assumed that the velocity profile in the laminar boundary layer upstream of the roughness has velocity components in the x and y directions only.

The above assumption is true if the lateral edge of the flat plate is sufficiently far from the roughness. The following notations are used for Cartesian coordinates and the respective vector components of velocity, total enthalpy, density, pressure, and dynamic viscosity:

$$xl,\ yl,\ zl,\ u_\infty u,\ u_\infty v,\ u_\infty w,\ \ u_\infty^2 H,\ \rho_\infty \rho,\ \rho_\infty u_\infty^2 p,\ \mu_\infty \mu$$

(the subscript ∞ denotes dimensional quantities in the free stream). Limiting flow situation at large but subcritical Reynolds numbers ($\mathrm{Re} = \rho_\infty u_\infty l/\mu_\infty$) when laminar flow is retained is considered.

It is assumed that the transverse dimension b of the roughness is of the same order as the streamwise dimension a [the flow near slender roughness $b = o(a)$ will be considered below]. It is worth mentioning that the flow past roughness with $a = o(b)$ reduces to a two-dimensional problem and the flow past roughness with equal streamwise and transverse dimensions leads to a three-dimensional problem. It is also assumed that for given values of a and b the thickness of the roughness c is such that in the disturbed flow near the roughness the order of the viscous force is not less than the inertial force. Considering the velocity profile in the boundary layer near the surface of the body to be $u \sim y/\delta_0$ and making an order of magnitude analysis of terms representing viscous and inertial forces in the x-momentum equation, we get

$$c \leqslant O(a\varepsilon^{1/3}),\ \delta_0 = \varepsilon = \mathrm{Re}^{-1/2}. \qquad (1.1)$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 28-36, January-February, 1985. Original article submitted September 19, 1983.

 0021-8944/85/2601-0024$09.50

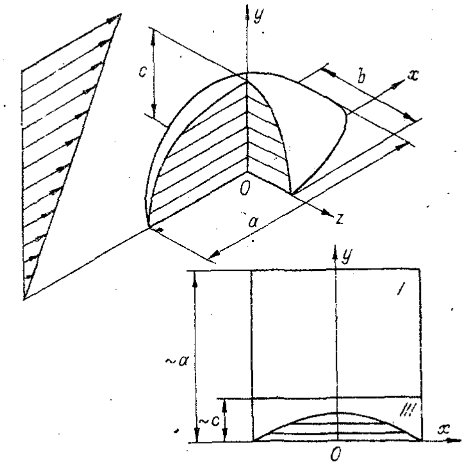

Fig. 1

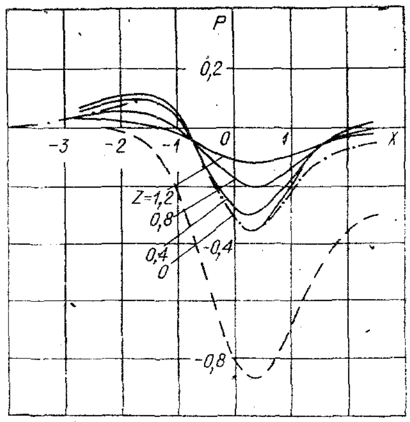

Fig. 2

In deriving Eq. (1.1) it was also assumed that the pressure gradient induced by the roughness is large ($\partial p/\partial x \gg 1$).

According to [1, 2], whose results for $a = O(b)$ can be generalized to three-dimensional flows, the structure of the flow near the roughness depends on the relative size a of the roughness with respect to the boundary-layer thickness $\delta_0 = O(\mathrm{Re}^{-1/2})$ ahead of the roughness. It follows from Eq. (1.1) that when $a > O(\varepsilon^{3/2})$ $c = o(a)$; hence, the maximum dimension of the disturbed flow region is determined by the dimensions a or b. If there is separation in the flow then it is assumed that its dimension also does not exceed the characteristic dimension a. The region with identical characteristic dimensions along x, y, and z axes and equal to a (region I) contains streamlines of the external inviscid irrotational flow, streamlines of the undisturbed boundary layer, or streamlines of the wall shear layer, if the streamwise dimension of the roughness a is an order of magnitude greater than the boundary-layer thickness δ_0, equal to this value, or asymptotically smaller, respectively. The flow in the region I happens to be weakly disturbed since the fulfillment of inequality (1.1) implies that nonlinear variations take place in the wall region III with thickness equal to the thickness of the roughness $c = o(a)$ for $a > O(\varepsilon^{3/2})$, i.e., at the bottom of the region I.

The solution to the boundary-value problem describing the flow in the region I makes it possible to establish a relation between the change in boundary-layer displacement thickness (or its wall sublayer thickness) and the induced change in pressure, which is known a priori, and should be determined during the simultaneous solution of the boundary-value problems in the characteristic regions of the flow. If the streamwise dimension of the roughness a exceeds the boundary-layer thickness δ_0, then for subsonic external flow, the boundary-value problem in the region I reduces to Laplace equation with homogeneous boundary conditions at large distances from the roughness or to hyperbolic equation of the linear supersonic flow theory. Contribution to the change in displacement thickness is made, in the general case, by thickness of the roughness as well as by the change in the thickness of the region with nonlinear variations in the flow near the roughness. Such a flow situation is considered in [10] for hypersonic external flow, assuming hypersonic interaction parameter to be small and in [11] for subsonic external flow. Flow past shorter roughness elements is studied in the present paper. It is shown in [1, 2] that if the streamwise dimension a of the roughness is less than $a_0 = O(\varepsilon^{3/4})$, when the contribution to the change displacement thickness due to the thickness of the roughness and due to the change in the thickness of the nonlinear wall region is the same, then "compensation" condition is realized. In such a situation there is no disturbance in the region I to the first approximation.

When $a > O(\delta_0)$, the disturbed flow consists of three regions with the thickness of the region II equal to the boundary-layer thickness ahead of the roughness. It appears that in view of the small changes in functions in the region II, the change in its thickness is an order of magnitude less than the change in the thickness of the near-wall region in the absence of intense cooling or strong suction. The role of the region II consists of transferring the change in thickness of the region III to the region I. In the "compensated" condition the increase or decrease in the thickness of the roughness is compensated by the corresponding reduction or increase in thickness of the nonlinear flow region, since the total thickness remains constant and there are no disturbances in regions I and II.

It follows from the results of [1, 2] that "compensation" condition leads to the condition

$$\frac{\partial u}{\partial x}(x, y_1 = 0, z) = 0, \quad y_1 = y/a. \tag{1.2}$$

In the region III whose characteristic dimensions coincide with the dimensions of the roughness, the following coordinates are introduced

$$(x, y, z) = (ax_3, \varepsilon a^{1/3} y_3, az_3) \tag{1.3}$$

and the functions are denoted by

$$(u, v, w, p, \rho, \mu) = \left(a^{1/3} u_3, \varepsilon a^{-1/3} v_3, a^{1/3} w_3, p_\infty/\rho_\infty u_\infty^2 + a^{2/3} p_3, \rho_w, \mu_w\right). \tag{1.4}$$

The substitution of Eqs. (1.3) and (1.4) in Navier–Stokes equations and taking the limiting condition

$$\varepsilon \to 0, \ a \to 0, \ b = O(a) \tag{1.5}$$

lead to the following system of equations

$$\begin{gathered}\rho_w u_3 \frac{\partial u_3}{\partial x_3} + \rho_w v_3 \frac{\partial u_3}{\partial y_3} + \rho_w w_3 \frac{\partial u_3}{\partial z_3} + \frac{\partial p_3}{\partial x_3} = \mu_w \frac{\partial^2 u_3}{\partial y_3^2}, \\ \rho_w u_3 \frac{\partial w_3}{\partial x_3} + \rho_w v_3 \frac{\partial w_3}{\partial y_3} + \rho_w w_3 \frac{\partial w_3}{\partial z_3} + \frac{\partial p_3}{\partial z_3} = \mu_w \frac{\partial^2 w_3}{\partial y_3^2}, \\ \frac{\partial u_3}{\partial x_3} + \frac{\partial v_3}{\partial y_3} + \frac{\partial w_3}{\partial z_3} = 0\end{gathered} \tag{1.6}$$

with boundary conditions

$$\begin{aligned} &u_3 = d_0 y_3, \ w_3 = 0 \quad \text{as} \quad x_3 \to -\infty, \\ &u_3 = w_3 = v_3 = 0 \quad \text{at} \quad y_3 = c\varepsilon^{-1} a^{-1/3} f(x_3, z_3), \\ &u_3 = d_0 y_3, \ w_3 = 0 \quad \text{as} \quad y_3 \to \infty. \end{aligned} \tag{1.7}$$

The pressure distribution $p_3(x_3, z_3)$ is initially unknown and in order to determine it an auxiliary boundary condition which is added to the function u_3 as $y_3 \to \infty$ $(u_3 - d_0 y_3 = \varphi(p_3))$ is used. In particular, condition (1.2) is equivalent to the condition $\varphi \equiv 0$. The substitution of variables in the boundary-value problem (1.6), (1.7)

$$\begin{gathered} y_3 = (\tilde{y}_3 + f)\, c a^{-1/3} \varepsilon^{-1}, \quad x_3 = \rho_w d_0 c^3 \varepsilon^{-3} a^{-1} \mu_w^{-1} \tilde{x}_3, \\ v_3 = \left(\tilde{v}_3 + \tilde{u}_3 \frac{\partial f}{\partial \tilde{x}_3} + \tilde{w}_3 \frac{\partial f}{\partial \tilde{z}_3}\right) c^{-1} \mu_w \varepsilon a^{1/3} \rho_w^{-1}, \quad u_3 = c d_0 \varepsilon^{-1} a^{-1/3} \tilde{u}_3, \\ z_3 = \rho_w d_0 c^3 \varepsilon^{-3} a^{-1} \mu_w^{-1} \tilde{z}_3, \quad w_3 = c d_0 \varepsilon^{-1} a^{-1/3} \tilde{w}_3, \quad p_3 = \rho_w c^2 d_0^2 \varepsilon^{-2} a^{-2/3} \tilde{p}_3 \end{gathered}$$

leads to the boundary-value problem of the type

$$\begin{gathered} \tilde{u}_3 \frac{\partial \tilde{u}_3}{\partial \tilde{x}_3} + \tilde{v}_3 \frac{\partial \tilde{u}_3}{\partial \tilde{y}_3} + \tilde{w}_3 \frac{\partial \tilde{u}_3}{\partial \tilde{z}_3} + \frac{\partial \tilde{p}_3}{\partial \tilde{x}_3} = \frac{\partial^2 \tilde{u}_3}{\partial \tilde{y}_3^2}, \\ \tilde{u}_3 \frac{\partial \tilde{w}_3}{\partial \tilde{x}_3} + \tilde{v}_3 \frac{\partial \tilde{w}_3}{\partial \tilde{y}_3} + \tilde{w}_3 \frac{\partial \tilde{w}_3}{\partial \tilde{z}_3} + \frac{\partial \tilde{p}_3}{\partial \tilde{z}_3} = \frac{\partial^2 \tilde{w}_3}{\partial \tilde{y}_3^2}, \\ \frac{\partial \tilde{u}_3}{\partial \tilde{x}_3} + \frac{\partial \tilde{v}_3}{\partial \tilde{y}_3} + \frac{\partial \tilde{w}_3}{\partial \tilde{z}_3} = 0, \\ \tilde{u}_3 = \tilde{y}_3, \quad \tilde{w}_3 = 0 \quad \text{as} \quad \tilde{x}_3 \to -\infty, \quad \tilde{u}_3 = \tilde{w}_3 = \tilde{v}_3 = 0 \quad \text{at} \quad \tilde{y}_3 = 0, \\ \tilde{u}_3 = \tilde{y}_3, \quad \tilde{w}_3 = 0 \quad \text{as} \quad \tilde{y}_3 \to \infty, \quad \tilde{u}_3 - \tilde{y}_3 = \varphi(\tilde{p}_3) + f(\tilde{x}_3, \tilde{z}_3). \end{gathered} \tag{1.8}$$

2. When the thickness of roughness is small in terms of Eq. (1.3), the boundary-value problem (1.8) can be linearized. The following expressions for functions are introduced:

$$\tilde{u}_3 = \tilde{y}_3 + \lambda U, \quad \tilde{v}_3 = \lambda V, \quad \tilde{p}_3 = \lambda P, \quad \tilde{w}_3 = \lambda W, \quad y_3 = Y, \quad x_3 = X, \tag{2.1}$$

where $f = \lambda F$, $\lambda \ll 1$. The boundary-value problem in terms of variables (2.1) has the form

$$Y \partial U/\partial X + V + \partial P/\partial X = \partial^2 U/\partial Y^2,$$

$$Y\partial W/\partial X + \partial P/\partial Z = \partial^2 W/\partial Y^2$$
$$\begin{aligned} &\partial U/\partial X + \partial V/\partial Y + \partial W/\partial Z = 0, \\ &U = F,\ W = 0 \quad \text{as} \quad Y \to \infty, \\ &U = W = 0 \quad \text{as} \quad X \to -\infty, \\ &U = W = V = 0 \quad \text{at} \quad Y = 0. \end{aligned} \qquad (2.2)$$

Differentiating the first equation of the boundary-value problem (2.2) with respect to variables X and Y and the second equation with respect to Z and Y after adding these equations leads to the following equation with the help of continuity equation

$$Y\partial s/\partial X = \partial^2 s/\partial Y^2 \qquad (2.3)$$

with boundary conditions for the function $s = -\partial^2 V/\partial Y^2$

$$\begin{aligned} &s = 0 \quad \text{as} \quad X \to -\infty,\ s = 0 \quad \text{as} \quad Y \to \infty; \\ &\partial s/\partial Y = \partial^2 P/\partial X^2 + \partial^2 P/\partial Z^2 \quad \text{at} \quad Y = 0, \\ &\int_0^\infty s dY = \frac{\partial}{\partial X}[\varphi(P) + F(X, Z)] \end{aligned} \qquad (2.4)$$

($\varphi \equiv 0$, if "compensated" flow condition is considered). Equation (2.3) for function s(X, Y, Z) is parabolic. Analysis of the last two boundary conditions (2.4) shows that the pressure distribution is described by elliptic equation.

The Navier–Stokes equations from which Eq. (2.3) is obtained are of elliptic type in functions u, v, w, and p. The transition as $\mathrm{Re} \to \infty$ to a system of equations for three-dimensional boundary layer leads to parabolic type of equations, as shown in [17]. The system of equations (2.3), (2.4) obtained from Navier–Stokes equations as a result of limiting transition contains elliptic equation only for the function P, i.e., this system in some sense occupies the intermediate place between the three-dimensional boundary-layer equations and Navier–Stokes equations.

In order to solve the boundary-value problem (2.2) it is possible to use Fourier transform in variables X and Z:

$$U_1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(X, Y, Z)\, e^{-i\alpha X - i\omega Z} dX dZ,$$

which gives

$$Yi\alpha U_1 + V_1 + i\alpha P_1 = U_1'',\ Yi\alpha W_1 + i\omega P_1 = W_1'',\ i\alpha U_1 + V_1' + i\omega W_1 = 0,$$
$$U_1(\alpha, Y, \infty) = F_1(\alpha, \omega),\ Y \to \infty.$$

Substitution of variables $Y = Y_1(i\alpha)^{-1/3}$ makes it possible to obtain for the function

$$f_1 = (i\alpha)^{1/3}(i\alpha U_1 + i\omega W_1)'$$

the equation

$$Y_1 f_1 = f_1'',$$

whose solution is Airy's function [18]

$$f_1 = c_1 \mathrm{Ai}(Y_1),$$

which is damped as $Y_1 \to \infty$. The following relation is obtained using boundary conditions

$$c_1 \int_0^\infty \mathrm{Ai}(Y_1)\, dY_1 = (i\alpha)^{4/3} F_1(\alpha, \omega),\quad c_1 \mathrm{Ai}'(0) = -P_1 \frac{\alpha^2 + \omega^2}{(i\alpha)^{1/3}},$$

which results in an expression for the function P_1

$$P_1 = -3\mathrm{Ai}'(0)(i\alpha)^{5/3} F_1/(\alpha^2 + \omega^2).$$

An equation for pressure fluctuation is obtained by inverse Fourier transform

$$P = -\frac{3\,\mathrm{Ai}(0)}{4\pi^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{(i\alpha)^{5/3} F_1(\alpha, \omega)\, e^{i\alpha X + i\omega Z}\, d\alpha d\omega}{\alpha^2 + \omega^2}.$$

For $F = \exp(-X^2 - Z^2)$ this integral reduces to the following:

$$P = -\frac{3\,\mathrm{Ai}'(0)}{\pi}\int_0^\infty\int_0^\infty \frac{r^{5/3}\, e^{-r^2/4-\omega^2/4}\cos\omega Z\cos\left(rX+\frac{5}{6}\pi\right)dr\,d\omega}{\omega^2+r^2}. \quad (2.5)$$

Computed pressure distributions P(X, Z) [see Eq. (2.5)] are shown in Fig. 2 where dashed line indicates pressure distribution for the plane flow near a roughness of the form $F = \exp(-X^2)$.

It is possible to mention that the transition to plane flow is accompanied by a redistribution of pressure, a decrease in pressure upstream and an increase downstream of the roughness. There is no pressure fluctuation upstream of the roughness in plane flow as $X < 0$ for finite roughness and only a change in the pressure level is permissible.

This fact has been established in [19] for hypersonic flow with strong interaction. "Compensated" flow condition assumes conservation of total thickness consisting of roughness and boundary-layer displacement thickness. In the plane flow, therefore, the appearance of pressure fluctuation is associated with the change in body thickness. This phenomenon is also explained by the absence of nonzero eigenvalues for the boundary-value problem (2.2) for $W = 0$, satisfying the condition for damping as $X \to -\infty$.

In three-dimensional flow the situation is completely different. In the absence of roughness the increase in pressure can be accompanied by cross flow so that the increase in displacement thickness due to increase in pressure can be compensated by decrease in displacement thickness due to cross flow. The system of equations (2.2) has, as shown in [13], eigenfunctions of the form $P = \sin\alpha Z \exp(\alpha X)$. Consequently, change in roughness shape along Z will lead to the appearance of velocity component W and with constant total displacement thickness it leads to a change in streamwise pressure which explains the possibility of upstream influence of pressure in three-dimensional "compensated" flow.

When new variables are introduced in the boundary-value problem (2.3)

$$X = X,\ Y = g(X)N,\ s = h(X)\psi(X,\ N,\ Z),$$

it takes the form

$$\psi'' = -g^2 g^{\cdot} N^2\psi' + \frac{g^3 h^{\cdot}}{h}N\psi + g^3 N\psi^{\cdot},\quad \psi(X,\infty,Z)=0;\ \int_0^\infty \psi\,dN = \frac{F^{\cdot}}{gh}, \quad (2.6)$$

where

$$\partial/\partial X = (\quad)^{\cdot} \text{ and } \partial/\partial N = (\quad)'.$$

For roughnesses of the type

$$F(X,\ Z) - \mathrm{const} \sim X^{(\beta+4)/3}$$

the boundary-value problem (2.6) reduces to a similarity solution

$$\psi'' + N^2\psi' - \beta N\psi = 0,\quad \psi(\infty) = 0,\quad \int_0^\infty \psi\,dN = 1, \quad (2.7)$$

then

$$\partial s/\partial Y|_{Y=0} \sim \psi'(0)X^{(\beta-1)/3}.$$

The boundary-value problem (2.7) is a typical problem with interaction. Usually such problems are solved by matching the boundary condition at $N = 0$ (see, e.g., [7]). However, the form of the second boundary condition (2.7) makes it possible to include it in the difference scheme and effectively use the shooting method (similar method was used in [20] to solve similar boundary-value problems).

It is obtained from the solution of problem (2.7) that when $\beta = -4$ the principal determinant of the difference scheme becomes zero. Obviously, this is the condition for the existence of nontrivial solution as $X \to -\infty$ for finite roughness elements (or in general for roughness with $F^{\cdot} \equiv 0$).

Numerical solution of the boundary-value problem (2.3), (2.4) was also obtained for roughness $F = e^{-X^2-Z^2}$. The pressure distribution along the line of symmetry of the roughness is shown for comparison in Fig. 2 by the dashed–dotted line. Certain difference in results is observed only away from the vertex of the roughness due to different means of specifying boundary conditions for the pressure P(X, Z) while carrying out analytical and difference computations.

3. It follows from the above results that the pressure distribution in the disturbed region is described by equations of the elliptic type. It is possible to use this equation to estimate transverse pressure gradient $\partial p/\partial z = O(ba^{-4/3})$. Since the order of the longitudinal pressure gradient $\partial p/\partial x$ is also preserved, pressure drop in the steamwise direction happens to be of the order of the quantity $O(ba^{-1/3})$. It is possible to estimate the transverse velocity component $w = O(ba^{-2/3})$ from transverse momentum equations.

A comparison of terms in the equation for the longitudinal momentum shows that to the first approximation the pressure gradient $\partial p/\partial x$ is absent in this equation. Thus, when $b = o(a)$ the original boundary-value problem is split into two. The first one describes the flow in the region with characteristic dimensions

$$y = O(c),\ x = O(a),\ z = O(b).$$

The system of equations for this region is given by system (1.8) in which the term $\partial p_3/\partial x_3$ is absent in the first equation. The solution of this system makes it possible to determine the pressure gradient $\partial p_3/\partial z_3$ ($z_3 = 0$).

The second problem describes the flow in the region with characteristic dimensions $x = O(a)$, $y = O(c)$, and $z = O(a)$.

It is worth noting that fluctuations in functions u, v happen to be less in this region than near the roughness. The flow here is described by the linear system of Eqs. (2.2) in which the pressure distribution is obtained by solving Eqs. (2.2) with boundary conditions for transverse pressure gradient $\partial p_3/\partial z_3|_{z_3=0}$ and velocity component $w_3(z_3 = 0)$, obtained from the solution in the region adjacent to the roughness.

In the limit as $b \to 0$ the transverse dimension and the thickness of the roughness may be of the same order $b = O(c)$. For such a flow condition the independent variables and flow functions can be expressed in the form

$$(x, y, z, u, v, w, p, \rho) = (a\bar{x}, \varepsilon a^{1/3}\bar{y}, \varepsilon a^{1/3}\bar{z}, a^{1/3}\bar{u}, \varepsilon^{-1/3}\bar{v}, \varepsilon a^{-1/3}\bar{w}, \varepsilon^2 a^{-2/3}\bar{p}, \rho_w). \tag{3.1}$$

The substitution of the series (3.1) in Navier–Stokes equations and the limiting transition (1.5) lead to the following system of equations

$$\begin{gathered}
\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} = \nu_w\left(\frac{\partial^2\bar{u}}{\partial\bar{y}^2} + \frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right),\\
\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}} + \frac{\partial\bar{p}}{\partial\bar{y}}\frac{1}{\rho_w} = \nu_w\left(\frac{\partial^2\bar{v}}{\partial\bar{y}^2} + \frac{\partial^2\bar{v}}{\partial\bar{z}^2}\right),\\
\bar{u}\frac{\partial\bar{w}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{w}}{\partial\bar{y}} + \bar{w}\frac{\partial\bar{w}}{\partial\bar{z}} + \frac{1}{\rho_w}\frac{\partial\bar{p}}{\partial\bar{z}} = \nu_w\left(\frac{\partial^2\bar{w}}{\partial\bar{y}^2} + \frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right),\\
\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} + \frac{\partial\bar{w}}{\partial\bar{z}} = 0, \quad \nu_w = \mu_w/\rho_w
\end{gathered} \tag{3.2}$$

with boundary conditions of the type

$$\begin{gathered}
\bar{u} = \bar{v} = \bar{w} = 0 \ \text{when}\ \bar{y} = c\varepsilon^{-1}a^{-1/3}f, \quad \bar{u} = d_0\bar{y}, \quad \bar{v} = \bar{w} = \bar{p} = 0,\\
\bar{y}^2 + \bar{z}^2 \to \infty,\\
\bar{u} = d_0\bar{y}, \quad \bar{w} = \bar{v} = 0, \quad \bar{x} \to -\infty.
\end{gathered}$$

This system of equations with different boundary conditions was obtained in [21] where incompressible laminar flow was studied in the neighborhood of the line of intersection of two planes.

It was assumed above that the thickness of roughness c is of the order $O(\varepsilon a^{1/3})$. Satisfaction of this condition ensured identical effect of viscous and inertial forces on the flow near the roughness. If $c = O(\varepsilon a^{1/3})$ the effect of viscous force becomes dominant and the effect of inertial force is felt in higher-order approximations. Depending on the relation between the width of the roughness and other parameters of the problem, the following flow conditions past roughness can be realized.

When $O(\varepsilon a^{1/3}) < b < O(a)$ the most characteristic dimension in the direction OY is the quantity equal to $\varepsilon a^{-1/3}$. The flow is described by the linear system of equations (2.2) in which there is no longitudinal pressure gradient.

When $b = O(\varepsilon a^{1/3})$ the characteristic dimensions in the disturbed region are identical along OY and OZ. The flow in this case is described by the linearized system of equations (3.2) which has the form

$$\begin{aligned} \bar{y}\partial\bar{u}_1/\partial\bar{x} + \bar{v}_1 &= \nu_w(\partial^2\bar{u}_1/\partial\bar{y}^2 + \partial^2\bar{u}_1/\partial\bar{z}^2), \\ \bar{y}\partial\bar{v}_1/\partial\bar{x} + \partial\bar{p}_1/\partial\bar{y}_3 &= \nu_w(\partial^2\bar{v}_1/\partial\bar{y}^2 + \partial^2\bar{v}_1/\partial\bar{z}^2), \\ \bar{y}\partial\bar{w}_1/\partial\bar{x} + \partial\bar{p}_1/\partial\bar{z} &= \nu_w(\partial^2\bar{w}_1/\partial\bar{y}^2 + \partial^2\bar{w}_1/\partial\bar{z}^2), \\ \partial\bar{u}_1/\partial\bar{x} + \partial\bar{v}_1/\partial\bar{y} + \partial\bar{w}_1/\partial\bar{z} &= 0, \end{aligned} \tag{3.3}$$

where $\bar{u} = \bar{y} + \lambda\bar{u}_1$, $\bar{v} = \lambda\bar{v}_1$, $\bar{w} = \lambda\bar{w}_1$, $\lambda \ll 1$,

$$f = c^{-1}\lambda\varepsilon a^{1/3}F,$$

with boundary conditions

$$\begin{aligned} &\bar{u}_1 = -F(\bar{x}, \bar{z}),\ \bar{v}_1 = \bar{w}_1 = 0 && \text{at } \bar{y} = 0, \\ &\bar{u}_1 = \bar{v}_1 = \bar{w}_1 = 0 && \text{as } \bar{y} \to \infty, \\ &\bar{u}_1 = \bar{v}_1 = \bar{w}_1 = 0 && \text{as } \bar{x} \to -\infty. \end{aligned} \tag{3.4}$$

A decrease in roughness width leads to the situation that when $O(c) < b < O(\varepsilon a^{1/3})$, the effect of inertial forces is only of the second order and the disturbed flow in the region with characteristic dimensions $x \sim O(a)$, $y \sim z \sim O(b)$ is described by the system of equations (3.3) and boundary conditions (3.4) with zero convective terms in the first three equations.

In the above three flow conditions roughness led to linear variations in the streamwise velocity. Finally, when $b = O(c)$, the height of the roughness and the characteristic dimension of the disturbed flow are identical along OY. The flow in this case is described by the system of equation of the type

$$\frac{\partial^2 u_0}{\partial y_0^2} + \frac{\partial^2 u_0}{\partial z_0^2} = 0, \quad \frac{\partial p_0}{\partial z_0} = \frac{\partial^2 w_0}{\partial y_0^2} + \frac{\partial^2 w_0}{\partial z_0^2},$$
$$\frac{\partial p_0}{\partial y_0} = \frac{\partial^2 v_0}{\partial y_0^2} + \frac{\partial^2 v_0}{\partial z_0^2}, \quad \frac{\partial u_0}{\partial x_0} + \frac{\partial v_0}{\partial y_0} + \frac{\partial w_0}{\partial z_0} = 0,$$

where

$$u_0 = u\varepsilon c^{-1}, \quad v_0 = \varepsilon ac^{-2}v, \quad w_0 = \varepsilon ac^{-2}w, \ x_0 = a^{-1}x, \quad y_0 = c^{-1}y, \quad z_0 = c^{-1}z,$$

with boundary conditions

$$\begin{aligned} &u_0 = w_0 = v_0 = 0 && \text{at } y_0 = f, \\ &u_0 = y_0,\ p_0 = v_0 = w_0 = 0 && \text{when } y_0^2 + z_0^2 \to \infty. \end{aligned}$$

Returning to the analysis of roughnesses with identical dimensions along OX and OZ, it is necessary to observe that when $a = O(\varepsilon^{3/2})$ it follows from Eq. (1.1) that characteristic dimensions of roughness in all directions are identical. The flow is then described by Navier–Stokes equations for incompressible flow.

An example of numerical solution to such a system of equations describing flow past roughness at the bottom of laminar boundary layer is given in [14]. Singularity in this system of equations due to a reduction in the number of parameters characterizing roughness is not considered here. Further studies are necessary to investigate such flow conditions and to obtain solutions to the above described boundary-value problems.

LITERATURE CITED

1. V. V. Bogolepov and V. Ya. Neiland, "Supersonic, viscous flow past small roughness on the surface of a body," Tr. Tsentralnyi Aero Gidrodin. Inst., 1363 (1974).
2. V. V. Bogolepov and V. Ya. Neiland, "Studies on local disturbances in viscous supersonic flows," in: Problems in Aerodynamics [in Russian], Nauka, Moscow (1975).

3. A. V. Zubtsov, "Influence of single roughness on boundary layer flow," Uch. Zap. Tsentral'nyi Aero Gidrodin. Inst., 2, No. 1 (1971).
4. F. T. Smith, "Laminar flows over a small hump on a flat plate," J. Fluid Mech., 57, 4 (1973).
5. V. V. Bogolepov, "Computation of supersonic boundary layer interaction with a thin obstacle," Uch. Zap. Tsentral'nyi Aero Gidrodin. Inst., 5, No. 6 (1974).
6. V. V. Bogolepov, "Flow past small cylindrical hump on a flat plate in supersonic viscous flow," Uch. Zap. Tsentral'nyi Aero Gidrodin. Inst., 6, No. 2 (1975).
7. D. O. Lyzhin, "Flow past a smooth hump by a supersonic viscous fluid with free interaction," Uch. Zap. Tsentral'nyi Aero Gidrodin. Inst., 7, No. 4 (1976).
8. A. V. Kazakov, "Numerical study of flow past small roughness on flat plate in supersonic flow," Tr. Tsentral'nyi Aero Gidrodin. Inst., 1910 (1978).
9. A. V. Kazakov, "Computation of flow past a small cavity and a hump on a flat plate in a supersonic viscous flow," Tr. Tsentral'nyi Aero Gidrodin. Inst., 1910 (1978).
10. I. I. Lipatov, "Three-dimensional flow past a small roughness in laminar boundary layer," Uch. Zap. Tsentral'nyi Aero Gidrodin. Inst., 11, No. 2 (1980).
11. F. T. Smith, R. I. Sykes, and P. W. M. Brighton, "A two-dimensional boundary layer encountering a three-dimensional hump," J. Fluid Mech., 83, 1 (1977).
12. R. I. Sykes, "Stratification effects in boundary-layer flow over hills," Proc. R. Soc. London, A 361, N 1705 (1978).
13. I. I. Lipatov, "Flow past locally three-dimensional roughness at the bottom of laminar boundary layer with weak interaction in hypersonic flow," Tr. Tsentral'nyi Aero Gidrodin. Inst., 2079 (1980).
14. P. J. Mason and R. I. Sykes, "Three-dimensional numerical integrations of the Navier—Stokes equations for flow over surface mounted obstacles," J. Fluid Mech., 91, 3 (1979).
15. F. T. Smith, "A three-dimensional boundary-layer separation," J. Fluid Mech., 99, 1 (1980).
16. R. I. Sykes, "On three-dimensional boundary layer flow over surface irregularities," Proc. R. Soc. London, A 373, N 1754 (1980).
17. Wang, "On the determination for three-dimensional boundary layer equations," J. Fluid Mech., 48, 2 (1971).
18. M. Abramovits and I. Stigan, Handbook on Special Functions [in Russian], Nauka, Moscow (1979).
19. K. Stewartson and P. G. Williams, "Self-induced separation," Proc. R. Soc. London, A 312, N 1509 (1969).
20. A. A. Kovalenko and V. Ya. Neiland, "Strong interaction in boundary layer with hypersonic flow with local disturbances in boundary conditions," Uch. Zap. Tsentral'nyi Aero Gidrodin. Inst., 6, No. 2 (1975).
21. S. G. Rubin and B. Grossman, "Viscous flow along a corner: numerical solution of the corner layer equations," Q. Appl. Math., 29, No. 2 (1971).